POSTGRADUATE SEMINAR SERIES - 1ST DECEMBER 2022

# Extremism, segregation and oscillatory states emerge through collective opinion dynamics in a novel agent-based model

# Overview

- What is Opinion Dynamics and where does our model come in?
- The model and a result on sufficient conditions for consensus.
- Numerical results and the emergent behaviours produced by the model.
- Conclusions and future research directions.

# Introduction

- What is Opinion Dynamics?
- In the original DeGroot model (1974), agents iteratively update their opinions according to a weighted average.
- Other types of model include: the Voter Model (Holley and Ligget, 1975); and Bounded Confidence models (Deffuant, 2000; Hegselmann and Krause, 2002).
- What sort of features have been incorporated into these models?
- Are we limited to these approaches?

# So where does our model fit in?

- Incorporates and extends elements from bounded confidence, voter and DeGroot-Friedkin models

- Novel features of the model include:
    - Agents can hold multiple opinions.
    - Opinions are continuous.
    - Affinity threshold now includes memory.

- Inspiration was also taken from models of collective animal motion, in particular the Cucker-Smale (2007) model of bird flocks.

# The Model: Key Notation

- Opinions are D-dimensional and evolved simultaneously.
- Euclidean distance from the origin is used as a measure of extremeness
- N agents evolve their D dimensional opinion vectors in discrete time
- Agents interact if their affinity exceeds their affinity threshold
- Affinities take values between 0 and 1 and must always be symmetric

$$a_{ij}(t) = \frac{1}{\left(1 + \sum_{\tau=0}^{t} w(\tau; t, \mu) \left\| \mathbf{v}_j(\tau) - \mathbf{v}_i(\tau) \right\|^2\right)^{1/2}},$$

$$w(\tau; t, \mu) = \begin{cases} 1, & \text{if } \tau > t - \mu, \\ 0, & \text{if } \tau \leq t - \mu, \end{cases}$$

# The Model: Opinion Updates

$$\text{For all } i: \mathbf{v}_i(t+1) = \mathbf{v}_i(t) + \frac{1}{Q_i(t;\rho_i)} \sum_{j=1}^{N} c_{ij}(t;\rho_i) a_{ij}(t) \left(\mathbf{v}_j(t) - \mathbf{v}_i(t)\right),$$

$$c_{ij}(t;\rho_i) = \begin{cases} 1, & \text{if } a_{ij}(t) > \rho_i, \\ 0, & \text{if } a_{ij}(t) \leq \rho_i, \end{cases}$$

$$Q_i(t;\rho_i) = \sum_{k=1}^{N} c_{ij}(t;\rho_i).$$

Two types of affinity threshold:

1. Universal threshold

$$\rho_i(t) = \rho$$

2. Individually evolving thresholds

$$\rho_i(t) = \rho + (1-\rho)\left(1 - e^{-\alpha \|\mathbf{v}_i(t)\|}\right).$$

# How can we guarantee consensus?

**Proposition 2.2.** Consider a population of agents $i = 1, 2, \ldots, N$, evolving their opinions $\mathbf{v}_i(t) \in \mathbb{R}^D$ according to the model (1)–(5), with some universal threshold $\rho_i(t) = \rho$ for all $i, t$.

(1) Given any initial condition, the opinions converges to some steady state: $\lim_{t\to\infty} \mathbf{v}_i(t) = \mathbf{v}_{*i}$ for all $i$.

(2) Given any initial condition and any $\rho < \rho_*$, where $\rho_*$ is given by (13) with $R_0 = \max_i\{\mathbf{v}_i(0)\}$, the opinions converge to a consensus: $\lim_{t\to\infty} \mathbf{v}_i(t) = \mathbf{v}_*$ for some common $\mathbf{v}_*$. Moreover,

$$\mathbf{v}_* = \frac{1}{N}\sum_{i=1}^{N} \mathbf{v}_i(0),$$

is the initial mean opinion of the population.

$$\rho_* = \frac{1}{(1 + 4\mu R_0^2)^{1/2}}$$

# Methodology

- N= 100 agents with initial opinions drawn from D independent normal distributions.
- 1000 sets of initial conditions for each parameter set.
- We say that a "cluster" has formed if all pairwise distances are less than 1e-6.
- The simulation has converged to a steady state if none or very little movement has occurred in 100 timesteps.

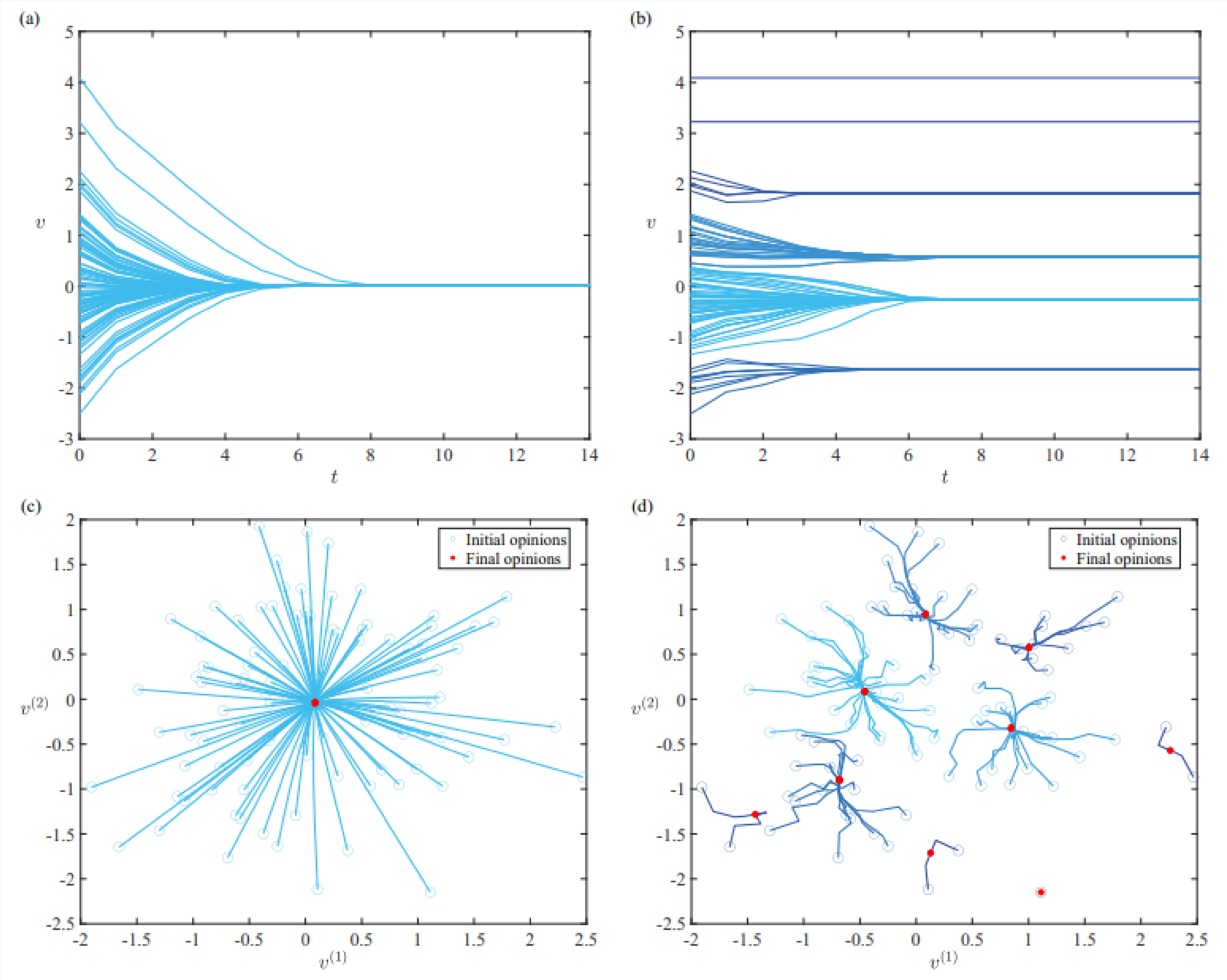
**Table 1.** Parameters used in numerical simulations.

| Parameter | Values used |
|---|---|
| $D$ (Dimensionality of every opinion) | 1, 2, 3, 5 |
| $\mu$ (Memory capacity of population) | 2, 10 |
| $\rho$ (Universal threshold in Section 3.2; baseline threshold in Section 3.3) | 0, 0.01, 0.02, . . . , 0.99 |
| $\alpha$ (Reinforcement rate; only in Section 3.3) | 0.1, 0.2, 0.4, 0.8 |

# Results

- Universal affinity thresholds
- Individually evolving affinity thresholds
- Failure to converge: collective oscillations

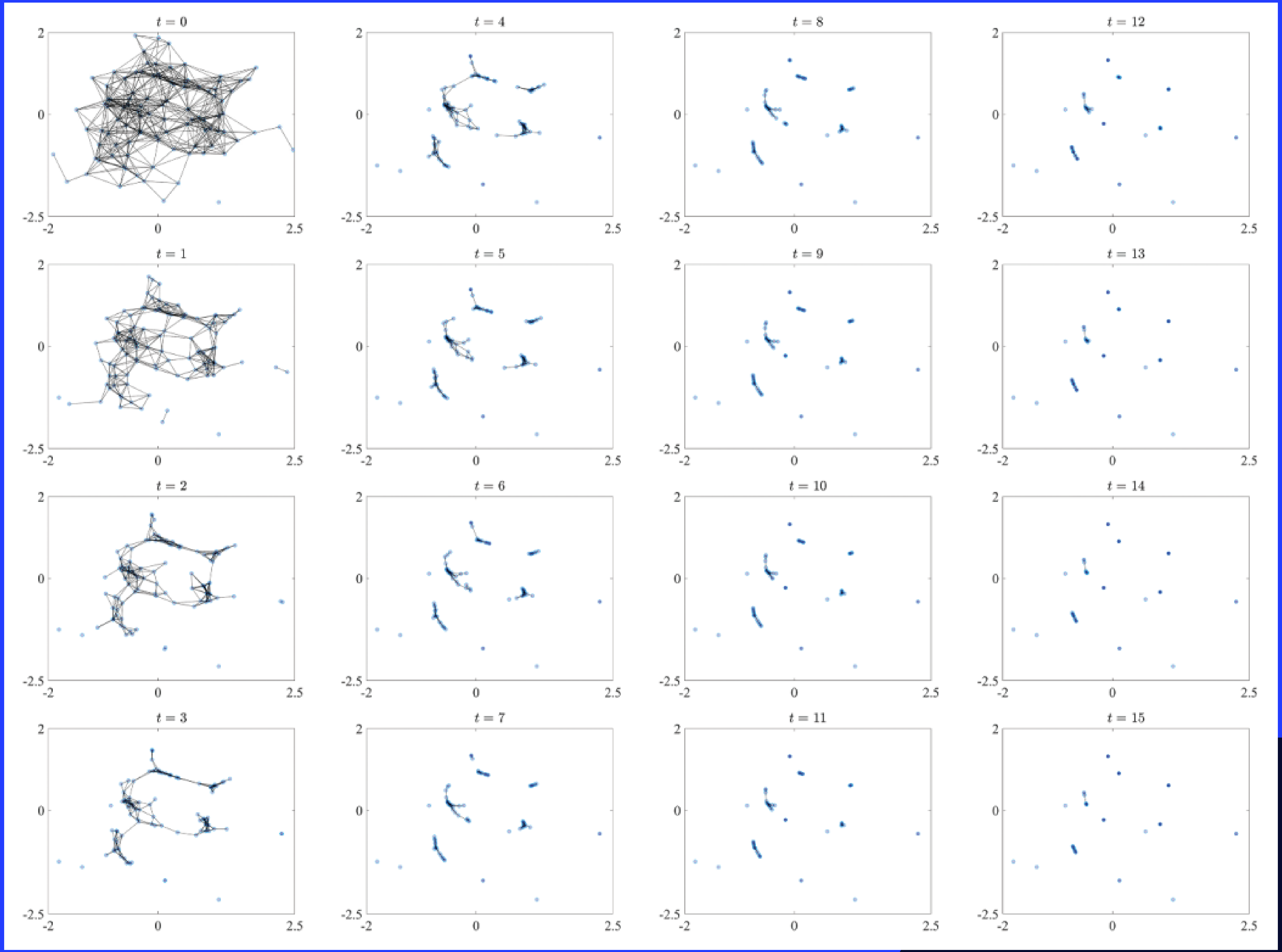
# Consensus




# Segregation

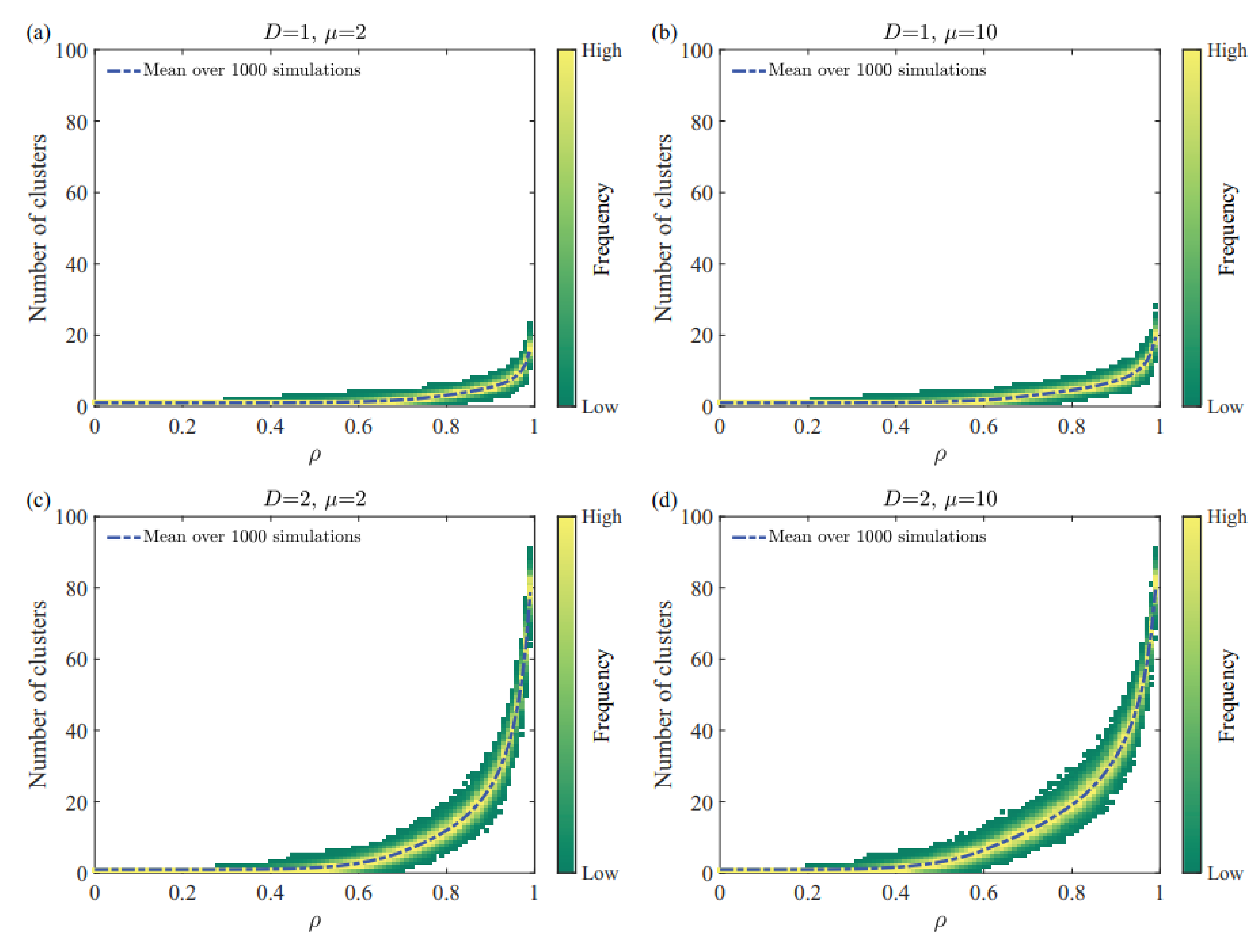
Universal Affinity Threshold

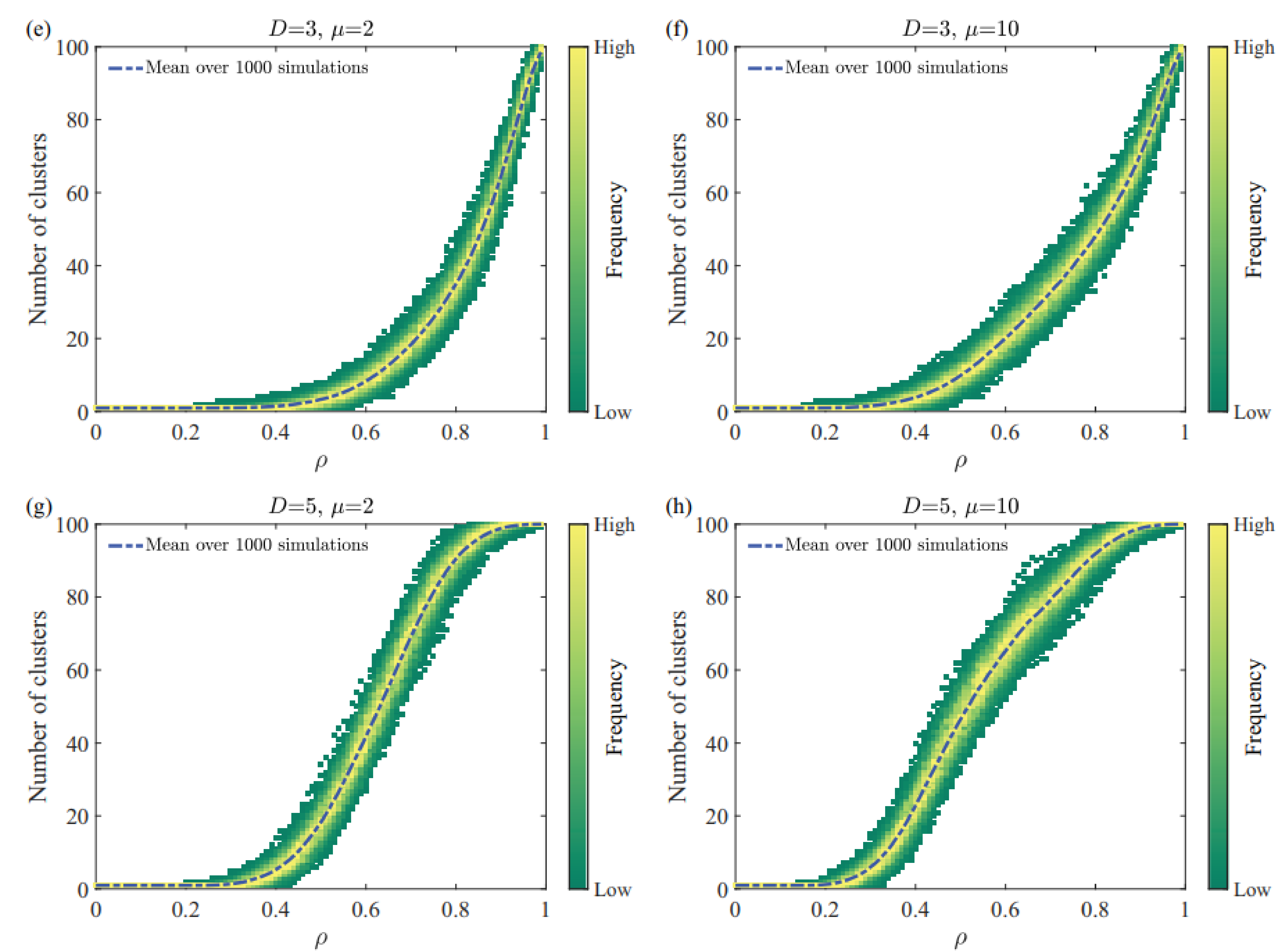
# Evolution of the Connectome




Universal Affinity Threshold

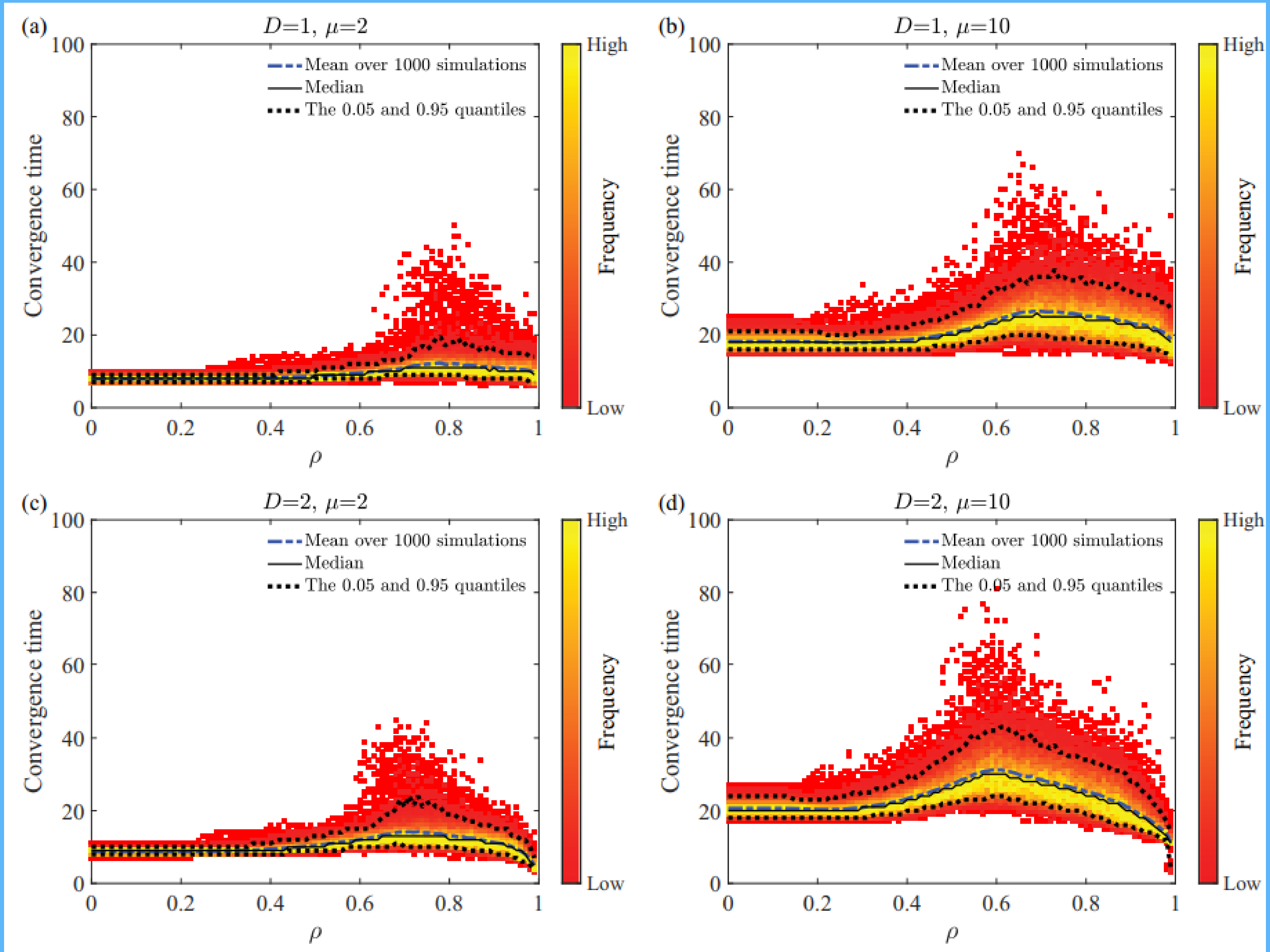
# Cluster Formation





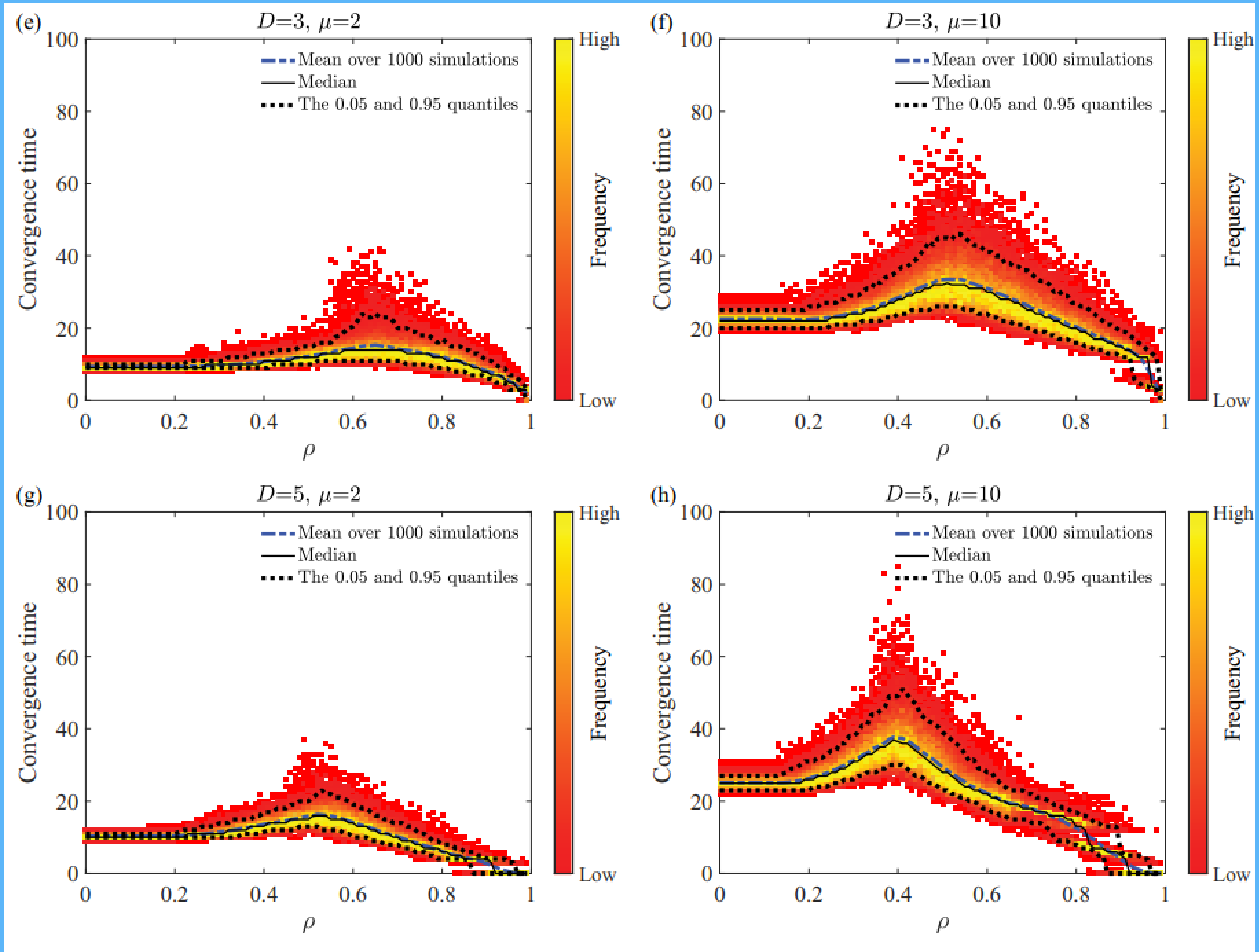

# Cluster Formation

# Convergence Time

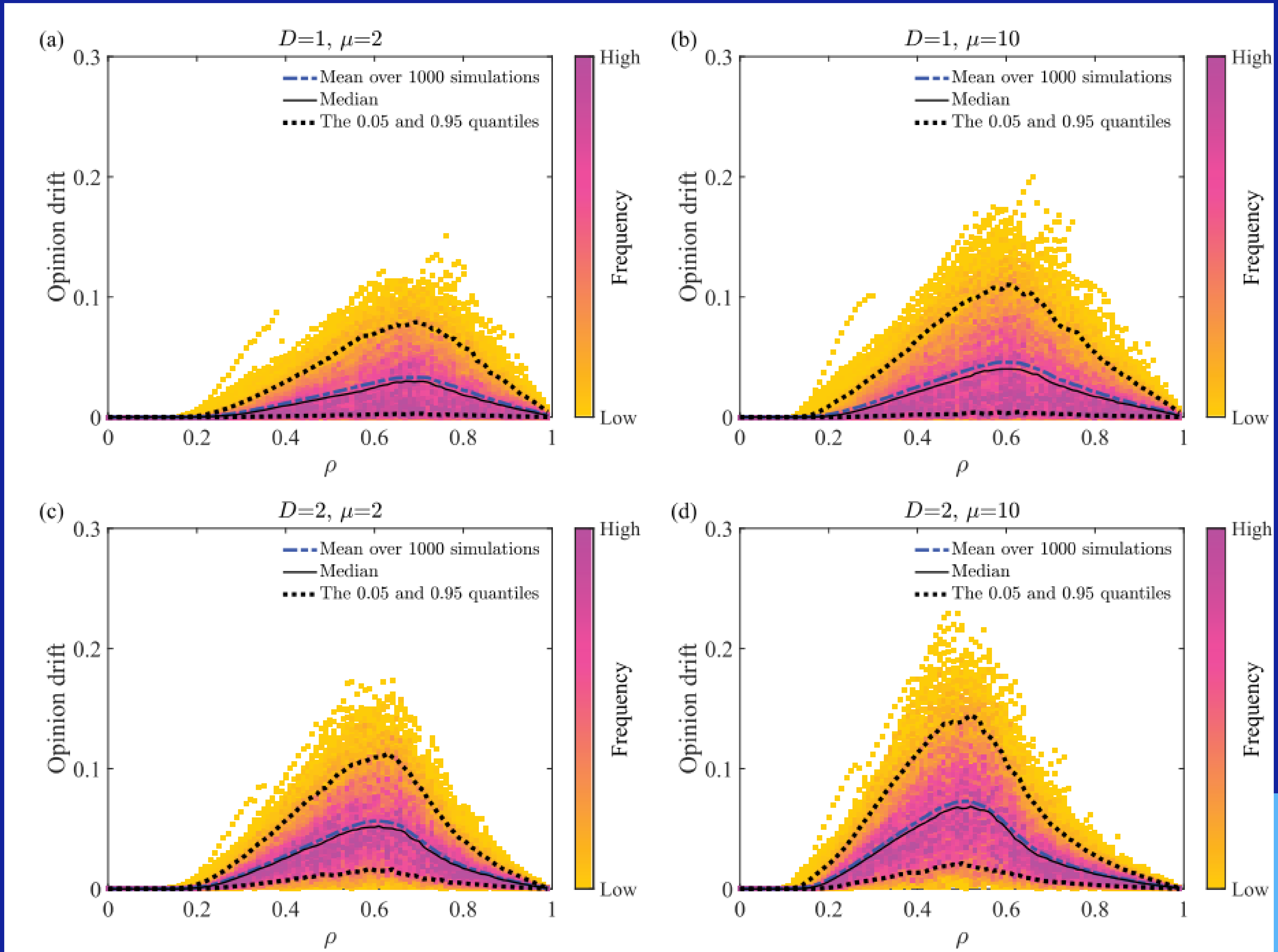



Universal Affinity Threshold

# Convergence Time

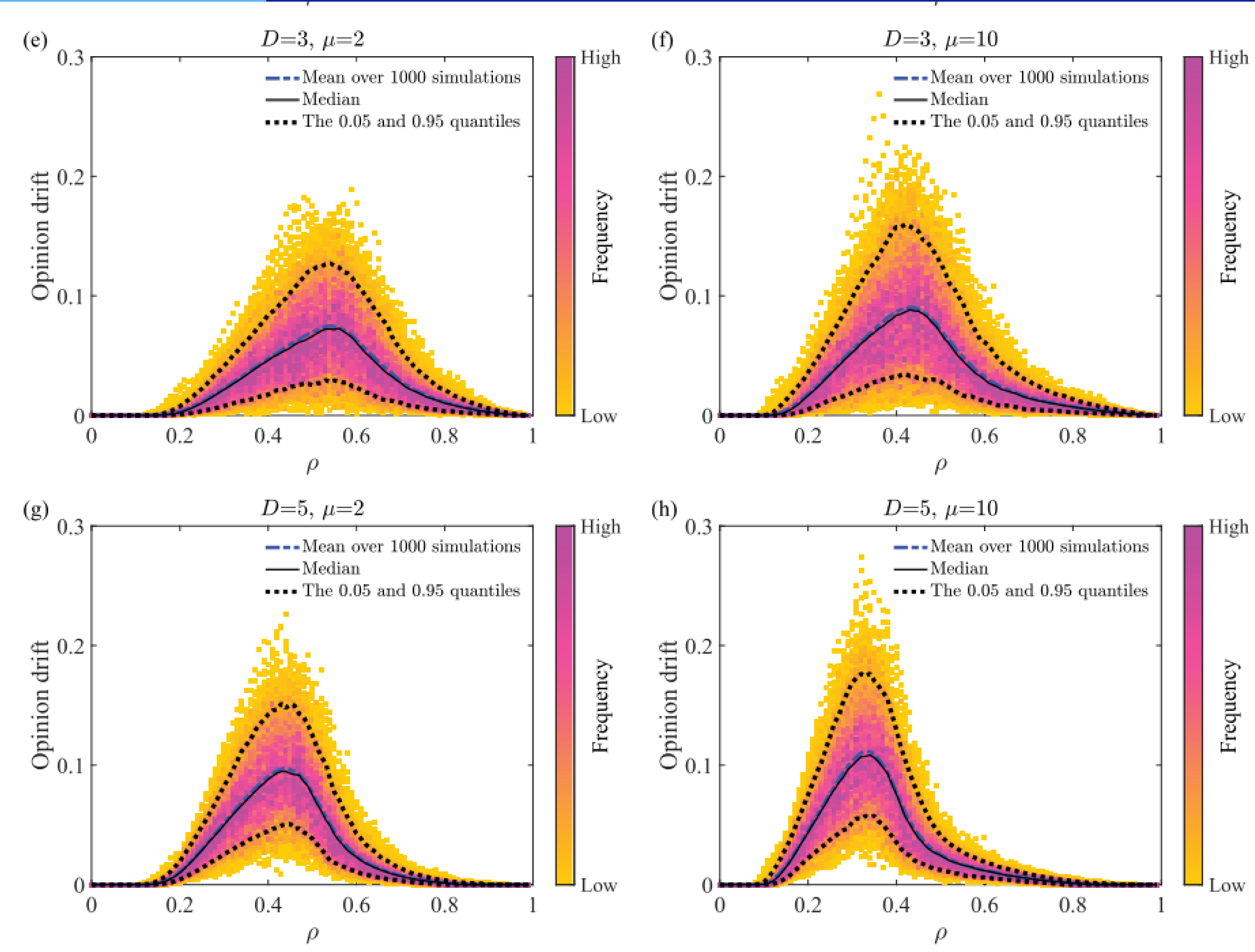


# Opinion Drift




Universal Affinity Threshold

# Opinion Drift



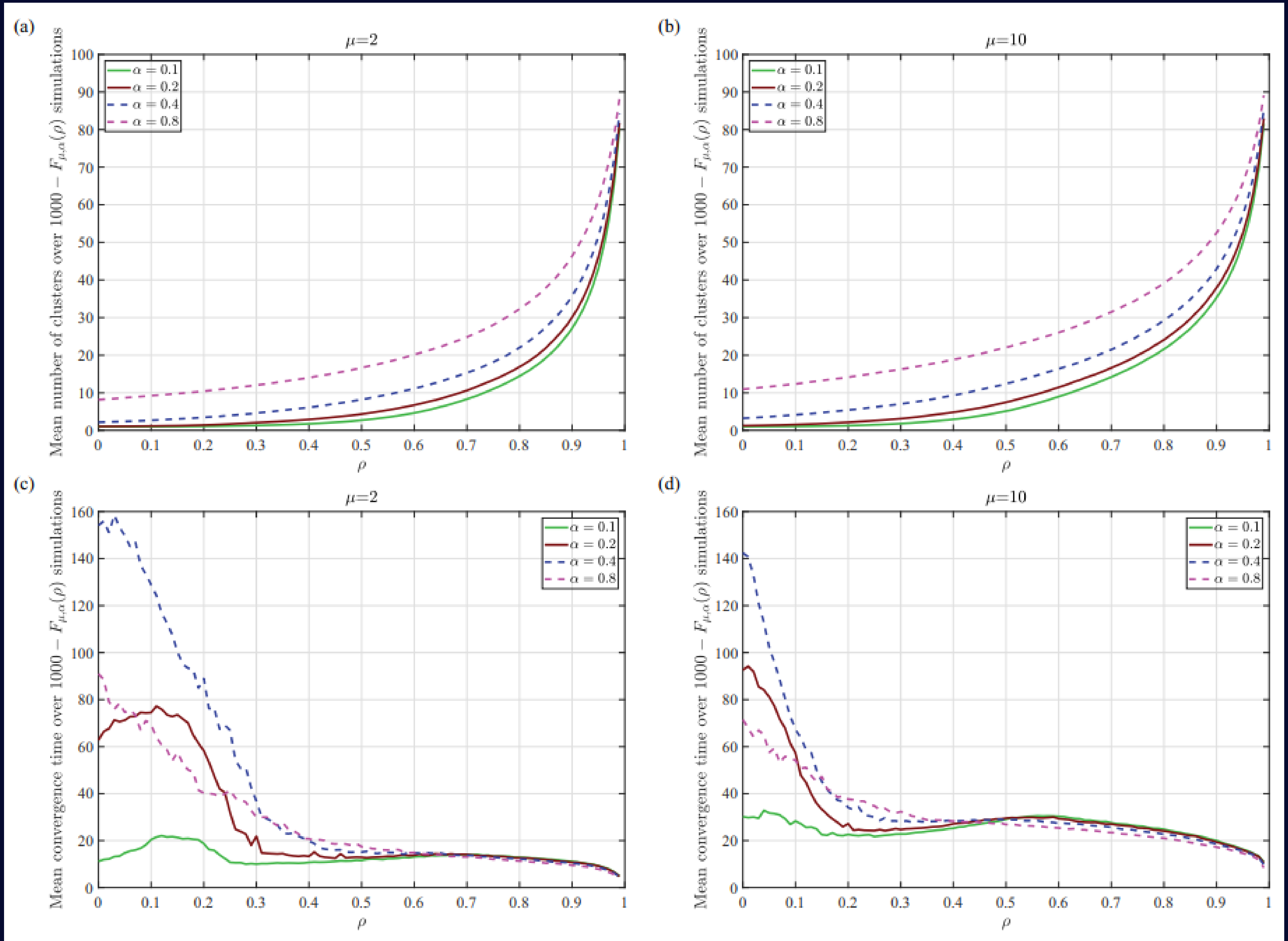

Universal Affinity Threshold

# Individually Evolving Affinity Thresholds

- We now fix D = 2 and allow agents to evolve their own affinity threshold.
- Vary the reinforcement rate, which determines how sharply affinity threshold increases with extremeness of opinion.
- Define an extremisation measure:

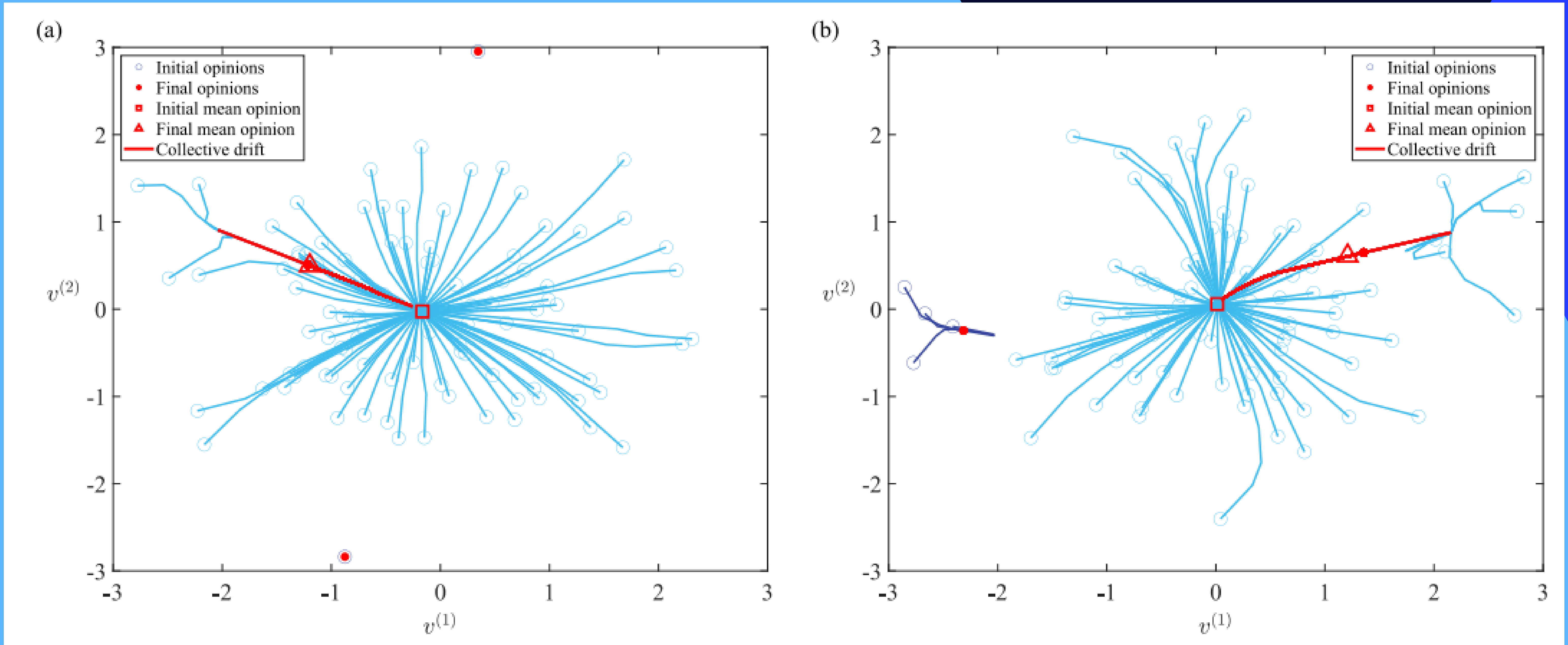
$$\text{Extremisation measure} = \left\| \frac{1}{N} \sum_{i=1}^{N} \mathbf{v}_i(t_c) \right\| - \left\| \frac{1}{N} \sum_{i=1}^{N} \mathbf{v}_i(0) \right\|$$

# Cluster Formation and Convergence Time



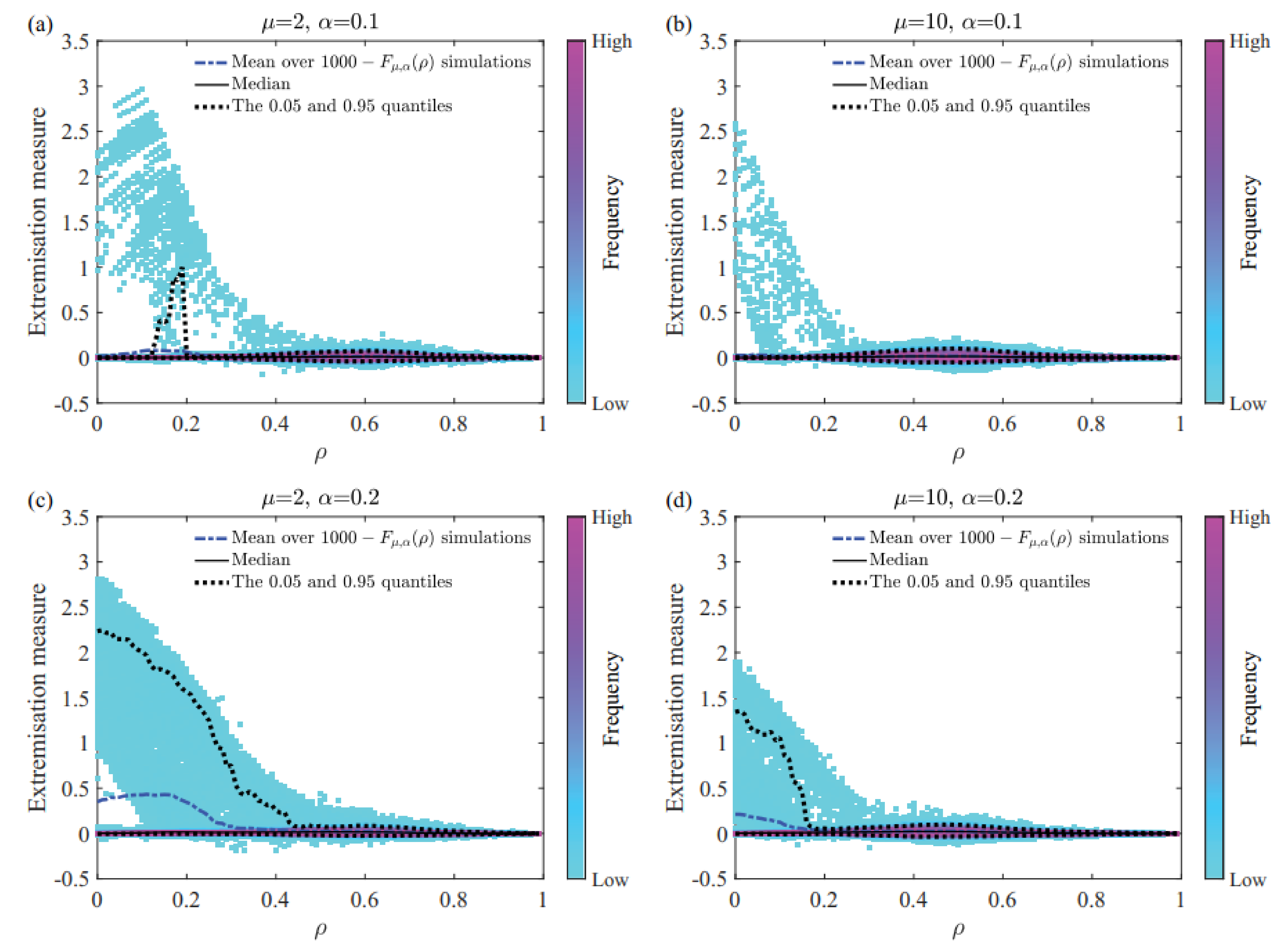

Individually Evolving Affinity Threshold

# Opinion Drift



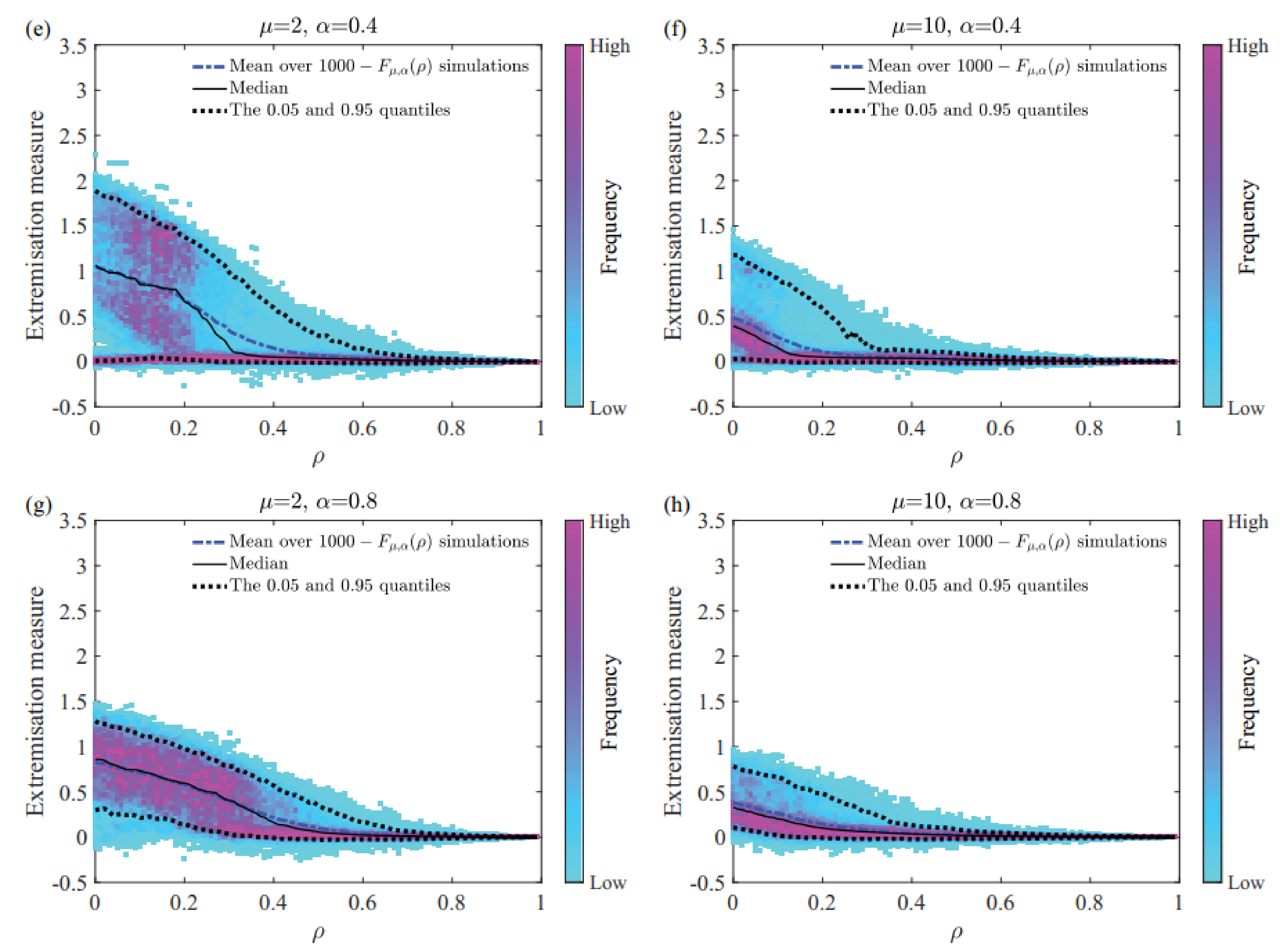

Individually Evolving Affinity Threshold

# Extremisation



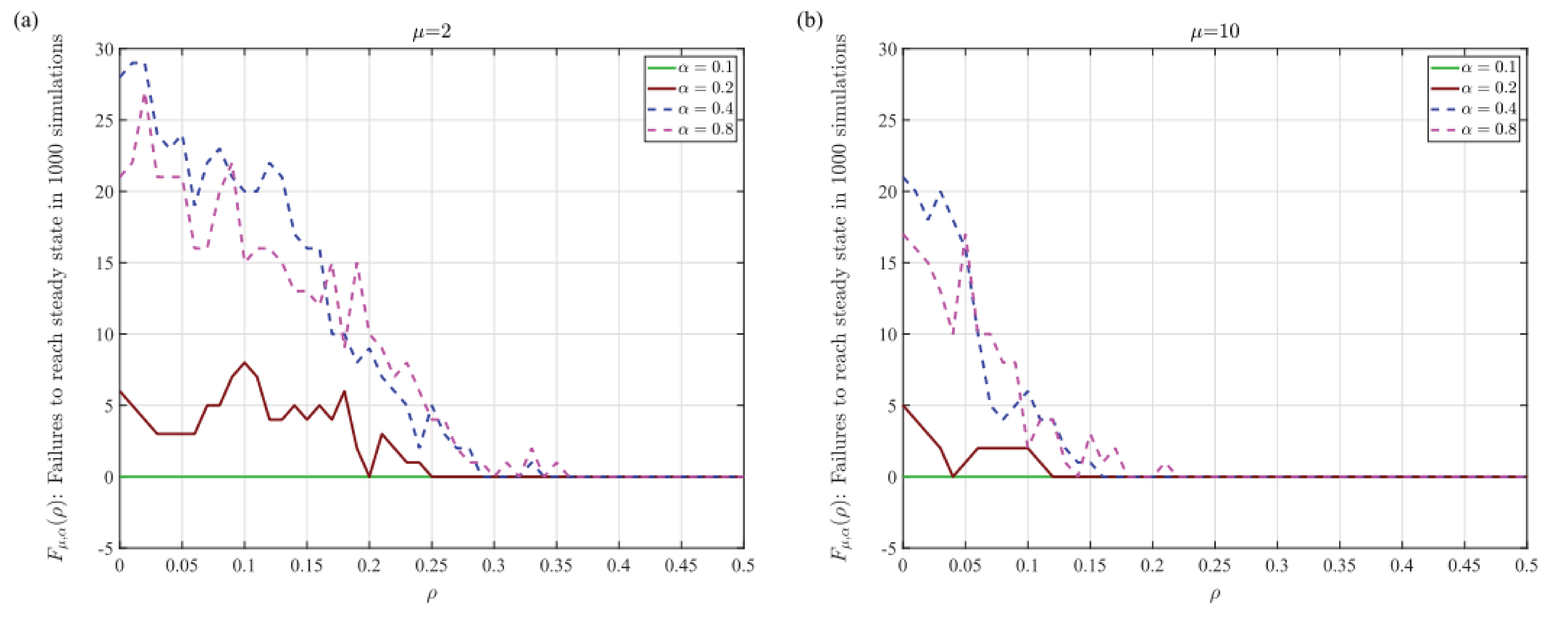

Individually Evolving Affinity Threshold

# Extremisation

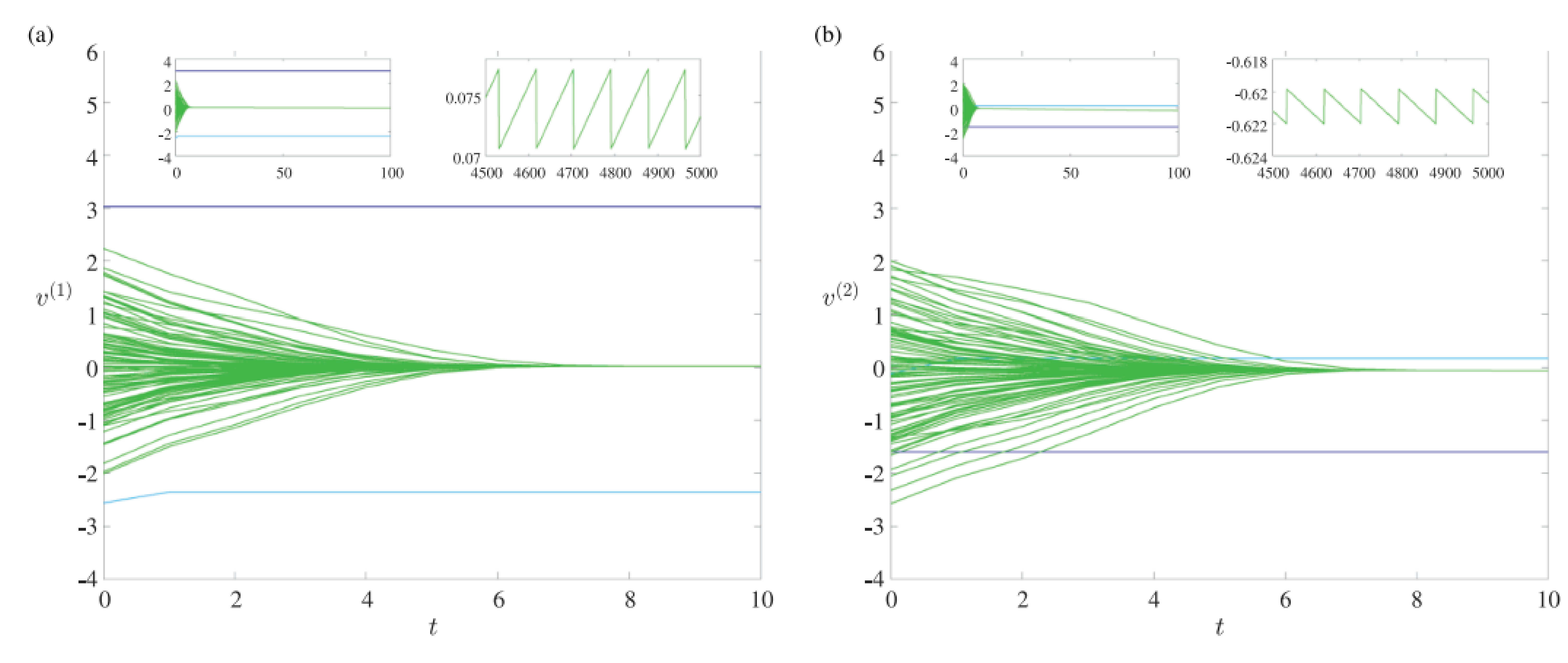



Individually Evolving Affinity Threshold

# Failure to converge: collective oscillations



Failure to converge: collective oscillations



# Conclusions

- Developed a novel model of opinion formation capable of mimicking socio-psychological phenomena such as emergent co-oporation and group polarisation.
- Extends existing theoretical findings and support experimental ones.
- Sufficiently low universal threshold guarantees consensus.
- A population which takes a longer history of itself into account is less susceptible to extremism.
- Heterogeneous networks permit oscillatory opinion clusters.

# Future Research?

- More sophisticated initial conditions, such as those that include correlations between opinions.
- Other potential extensions could include:
  - Hierarchical populations
  - Repulsive interactions
  - Stochastic fluctuations

# Thanks for listening!

## Any Questions?